General Gauge Mediation

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We give a general definition of gauge mediated supersymmetry breaking which encompasses all the known gauge mediation models. In particular, it includes both models with messengers as well as direct mediation models. A formalism for computing the soft terms in the generic model is presented. Such a formalism is necessary in strongly-coupled direct mediation models where perturbation theory cannot be used. It allows us to identify features of the entire class of gauge mediation models and to distinguish them from specific signatures of various subclasses.

§1. Introduction

Gauge mediation is one of the oldest, simplest, and most robust ways of transmitting SUSY breaking to the MSSM. It has a number of virtues, for instance guaranteeing flavor universality among the MSSM sfermion masses (thus solving the SUSY flavor problem). Unfortunately, even with the inherent simplicity of gauge mediation there is a veritable cornucopia of models (for a review of various types of gauge mediated models and some of the early history see Ref. 1)). These models have a wide variety of features, and often it is unclear which features are model specific and which are generic to gauge mediation. Furthermore, despite this long list of models, it is not obvious that all the possibilities of gauge mediation have been completely mapped out. For instance, as was originally envisioned,[2)] direct mediation models can be strongly coupled. Such models have not yet been extensively studied, in part because there is currently no developed framework for calculating the MSSM soft masses (for an early work see Ref. 3)).

In this paper we wish to address these points by presenting a unified framework to describe the effects of a completely arbitrary hidden sector. At the heart of this framework is a careful definition of the gauge mediation mechanism itself: *in the limit that the MSSM gauge couplings $\alpha_i \to 0$, the theory decouples into the MSSM and a separate hidden sector that breaks SUSY.* (In §5 we will slightly extend this definition to include various couplings to the Higgs field.) Here are some examples:

1. The most common paradigm of gauge mediation is to have a set of weakly coupled messenger fields charged under the MSSM and some supersymmetry breaking spurion field X.[4)–6)] Such models fit our definition by identifying the hidden sector as including both the supersymmetry breaking sector and the messengers; together, these decouple as $\alpha_i \to 0$. Clearly, we can accommodate any number of messengers and X fields. Also, various models with additional gauge field messengers which have independent gauge coupling constants (such as models with extra $U(1)$'s) can also be accommodated by including these fields in the hidden sector. Note however that gauge messenger models based

on nontrivial embeddings of the SM gauge group into larger groups such as $SU(5)_{GUT}$ (see e.g. Refs. 7)–10) and more recently Ref. 11)) are not covered by our formalism, because in these models the heavy gauge fields cannot be included in an almost decoupled hidden sector.

2. Models such as Refs. 12)–17) and more recently Ref. 18) involve a weakly coupled supersymmetry breaking theory (i.e. an O'Raifeartaigh-like model) with a global symmetry. These are direct mediation models, where the messenger fields participate in the supersymmetry breaking process.

3. Direct mediation models which involve a strongly coupled hidden sector (for a sample of such models, see Ref. 1)). Here, there may not even be identifiable messenger fields but the model still lies within our definition of gauge mediation.

Given this definition of gauge mediation which includes strongly coupled theories, the computation of the soft terms in the MSSM can proceed in perturbation theory in α_i but must include exact information from the hidden sector. This information is summarized in a set of correlation functions of real linear superfields $\mathcal{J}$ representing the hidden sector contribution to the gauge currents of the MSSM. In this framework it turns out that all the soft terms of the MSSM are describable in terms of only a small number of current correlation functions for any model of gauge mediation.

We do not provide a new toolkit for being able to calculate these current correlation functions for an arbitrarily strongly coupled theory. Nevertheless given any model for a hidden sector these correlation functions parameterize the answer for the effects of the hidden sector on the MSSM.

Superficially, our treatment is reminiscent of the effects described in Ref. 19) which looked at the influence of hidden sector running on the MSSM. However in Ref. 19), these effects are described by two scales: a scale where one integrates out some heavy messenger particles which couple the SUSY-breaking sector to the MSSM, and a lower scale (presumably the scale of SUSY breaking) where the rest of the hidden sector decouples. From our perspective of gauge mediation there is no weakly coupled description needed anywhere and there could in principle be only one scale.

With the framework of representing all the effects of gauge mediation in terms of current correlation functions we are able to derive the most generic predictions for gauge mediation. These include:

- Flavor universality among the sfermion masses,
- Sum rules for sfermions $\mathrm{Tr}\, Ym^2 = 0$ and $\mathrm{Tr}\,(B-L)m^2 = 0$ (with nonzero FI term for hypercharge, these sum rules are appropriately modified as shown in §4),
- Small A terms,
- Gravitino LSP.

Additionally there are several properties that can be true in a large set of models when more assumptions are made, but are not necessarily predictions of gauge mediation:

- Gaugino mass unification,
- Large hierarchies among sfermions with different gauge quantum numbers,
- A bino or stau NLSP.

It is important to point out that our framework — as we have presented it so far — does not allow for additional interactions which could generate μ and $B\mu$ radiatively. Since $U(1)_{PQ}$ needs to be broken to generate $\mu/B\mu$, it is necessary to introduce interactions between the MSSM and the hidden sector which remain even in the limit that the gauge couplings are turned off. In §5 we will present some preliminary remarks about how one could extend our general framework to include direct couplings between operators in the hidden sector and the Higgs fields of the MSSM. A successful solution to the $\mu/B\mu$ problem can then be characterized as certain conditions that the correlators of these operators must satisfy. We will save a more detailed analysis of this extended framework for future work.

The outline of our paper is as follows. In §2 we describe the global currents in the hidden sector and their relevant correlation functions. In §3 we weakly gauge the global symmetry and identify it with the SM gauge symmetry. We derive explicit formulas for the MSSM soft masses in terms of the current correlation functions. Section 4 contains our completely general derivation of the $U(1)_Y$ and $U(1)_{B-L}$ sum rules for gauge mediation. We also discuss various corrections to these sum rules from the unknown $\mu/B\mu$ sector, MSSM RG evolution, and electroweak symmetry breaking. Finally §5 contains a preliminary discussion of how to extend our formalism to include the sector that generates μ and $B\mu$, and how to phrase the μ problem of gauge mediation in this new language. In the Appendix we show how the standard analysis of models with messengers fits into our general framework.

§2. Currents in the hidden sector

In this section, we will work out expressions for the currents in the hidden sector and their correlation functions. For simplicity, we will only consider the case where a $U(1)$ is weakly gauged; the generalization to nonabelian groups is straightforward. Our conventions throughout will be chosen to agree with those of Ref. 20).

To begin, let us recall that the gauge current superfield

$$\mathcal{J} = \mathcal{J}(x, \theta, \overline{\theta}) \tag{2·1}$$

is a real linear superfield defined by the current conservation conditions

$$\overline{D}^2 \mathcal{J} = D^2 \mathcal{J} = 0 \ . \tag{2·2}$$

In components, it looks like

$$\mathcal{J} = J + i\theta j - i\overline{\theta}\overline{j} - \theta\sigma^\mu\overline{\theta} j_\mu + \frac{1}{2}\theta\theta\overline{\theta}\overline{\sigma}^\mu\partial_\mu j - \frac{1}{2}\overline{\theta}\overline{\theta}\theta\sigma^\mu\partial_\mu\overline{j} - \frac{1}{4}\theta\theta\overline{\theta}\overline{\theta}\Box J \tag{2·3}$$

with j_μ satisfying the condition

$$\partial^\mu j_\mu = 0. \tag{2·4}$$

Current conservation and Lorentz invariance imply that the only nonzero current one-point function is

$$\langle J(x) \rangle = \zeta\,. \tag{2·5}$$

(Obviously, ζ vanishes when one generalizes from $U(1)$ to nonabelian groups.) Meanwhile, the only nonzero current-current correlators are*)

$$\begin{aligned}
\langle J(x) J(0) \rangle &= \frac{1}{x^4} C_0(x^2 M^2)\,, \\
\langle j_\alpha(x) \bar{j}_{\dot\alpha}(0) \rangle &= -i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \left(\frac{1}{x^4} C_{1/2}(x^2M^2) \right)\,, \\
\langle j_\mu(x) j_\nu(0) \rangle &= (\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu) \left(\frac{1}{x^4} C_1(x^2M^2) \right)\,, \\
\langle j_\alpha(x) j_\beta(0) \rangle &= \epsilon_{\alpha\beta} \frac{1}{x^5} B_{1/2}(x^2M^2)\,.
\end{aligned} \tag{2·6}$$

Here M is some characteristic mass scale of the theory. The function $B_{1/2}(x^2M^2)$ is complex in general, but ζ and the functions $C_a(x^2M^2)$ must be real.

If supersymmetry is unbroken, we readily obtain

$$C_0 = C_{1/2} = C_1, \qquad B_{1/2} = 0. \tag{2·7}$$

The correlators are all finite in position space, and their short distance behavior is controlled by dimensional analysis. In particular, the functions C_a and $B_{1/2}$ are regular as $x \to 0$. Their small x behavior is determined by the operator product expansion and the UV dimensions of the operators. Since the identity operator always appears in the OPE of $\mathcal{O}(x)^\dagger \mathcal{O}(0)$, the short distance behaviors of C_a are

$$\lim_{x\to 0} C_0(x^2M^2) = \lim_{x\to 0} C_{1/2}(x^2M^2) = \lim_{x\to 0} C_1(x^2M^2) = c\,. \tag{2·8}$$

(As we will see in the next section, when the global symmetry is weakly gauged, c essentially corresponds to the change in the beta function of the gauge coupling.) Since our theory spontaneously breaks supersymmetry, its short distance behavior is supersymmetric, as in (2·7), and hence the constant c is the same for all three correlators. Since the correlator $\langle J(x)J(0)\rangle$ must be positive,

$$c > 0. \tag{2·9}$$

*) The correlator $\langle j_\mu(x) J(0) \rangle$ can be nonzero only if the global symmetry is spontaneously broken, a scenario we will not consider. To see that, note that current conservation requires it to be proportional to $\partial_\mu x^{-2}$. This behavior corresponds to an exchange of the Goldstone boson of the spontaneously broken symmetry.

All of this is to be contrasted with the OPE $j_\alpha(x)j_\beta(0)$ which does not include the identity operator (one easy way to see this is using the R-symmetry), so the leading singularity in the OPE must have the form

$$j_\alpha(x)j_\beta(0) \sim \epsilon_{\alpha\beta}x^{\Delta-5}\mathcal{O}+\dots \qquad (2\cdot10)$$

for some operator $\mathcal{O}$ with dimension $\Delta > 1$. (In fact, a stronger inequality can be proven using supersymmetry.) Therefore,

$$\lim_{x\to 0} B_{1/2}(x^2M^2) = 0. \qquad (2\cdot11)$$

The correlators in (2·6) can receive contributions from contact terms at $x = 0$. These depend on the regularization scheme and the precise definition of the theory. In our case, where we plan to gauge the global symmetry associated with this current, the contact terms are determined. These are easily obtained in momentum space as follows.

The (Euclidean) Fourier transforms of (2·6) are

$$\begin{aligned}
\langle J(p)J(-p)\rangle &= \tilde{C}_0(p^2/M^2;\, M/\Lambda)\,,\\
\langle j_\alpha(p)\overline{j}_{\dot\alpha}(-p)\rangle &= -\sigma^\mu_{\alpha\dot\alpha}p_\mu\tilde{C}_{1/2}(p^2/M^2;\, M/\Lambda)\,,\\
\langle j_\mu(p)j_\nu(-p)\rangle &= -(p^2\eta_{\mu\nu}-p_\mu p_\nu)\tilde{C}_1(p^2/M^2;\, M/\Lambda)\,,\\
\langle j_\alpha(p)j_\beta(-p)\rangle &= \epsilon_{\alpha\beta}M\tilde{B}_{1/2}(p^2/M^2)\,,
\end{aligned} \qquad (2\cdot12)$$

where a factor of $(2\pi)^4\delta^{(4)}(0)$ is understood, and Λ is a UV cutoff regulating the integrals

$$\begin{aligned}
\tilde{C}_a(p^2/M^2;\, M/\Lambda) &= \int d^4x\, e^{ipx}\frac{1}{x^4}C_a(x^2M^2)\,,\\
M\tilde{B}_{1/2}(p^2/M^2) &= \int d^4x\, e^{ipx}\frac{1}{x^5}B_{1/2}(x^2M^2)\,.
\end{aligned} \qquad (2\cdot13)$$

Here the functions $\tilde{C}_a$ are again real, while $\tilde{B}_{1/2}$ can be complex. As in (2·7), if SUSY is unbroken, $\tilde{C}_0 = \tilde{C}_{1/2} = \tilde{C}_1$, $\tilde{B}_{1/2} = 0$. Because of (2·8), the Λ dependence is

$$\tilde{C}_a(p^2/M^2;\, M/\Lambda) = 2\pi^2 c\log(\Lambda/M) + \text{finite}\,, \qquad (2\cdot14)$$

where when supersymmetry is broken the finite part depends on a. On the other hand, we have also indicated in (2·12) that $\tilde{B}_{1/2}$ is cutoff independent; this immediately follows from (2·11).

In writing (2·12) we have assumed a specific choice of contact terms at $x = 0$. They are set such that the currents satisfy the conservation equations in momentum space. This choice is motivated by our intention to gauge this global symmetry. More specifically, the contact terms which are proportional to delta functions in coordinate space are polynomial in the momentum which are arranged such that the form of (2·12) is valid.

§3. Gaugino and sfermion masses

Now let us weakly gauge the global symmetry of the previous section, by coupling the currents to a vector superfield,

$$\mathcal{L}_{int} = 2g \int d^4\theta \mathcal{J}\mathcal{V} + \cdots = g(JD - \lambda j - \overline{\lambda j} - j^\mu V_\mu) + \dots , \tag{3·1}$$

where we have used the Wess-Zumino gauge. The ellipses in (3·1) represent $\mathcal{O}(g^2)$ terms including two gauge fields. Such terms are necessary for gauge invariance.

Our hidden sector can be strongly coupled, in which case we cannot use perturbation theory. However, we can still assume that $g \ll 1$ and expand the functional integral to second order in g. The hidden sector contribution is captured by the two point functions (2·6) or (2·12). Terms of order g^2 in (3·1) lead to contact terms which are set such that the functional form in (2·12) is valid. (A familiar example is the seagull term in scalar electrodynamics.)

The effective Lagrangian for the gauge supermultiplet is

$$\begin{aligned}\delta\mathcal{L}_{eff} = \frac{1}{2}g^2\tilde{C}_0(0)D^2 - g^2\tilde{C}_{1/2}(0)i\lambda\sigma^\mu\partial_\mu\overline{\lambda} - \frac{1}{4}g^2\tilde{C}_1(0)F_{\mu\nu}F^{\mu\nu} \\ - \frac{1}{2}g^2(M\tilde{B}_{1/2}(0)\lambda\lambda + \text{c.c.}) + \dots ,\end{aligned} \tag{3·2}$$

where the ellipses represent terms with higher powers of momentum. From this expression, we see that $\tilde{C}_a$ correspond to wavefunction renormalizations of D, λ and A, while $\tilde{B}_{1/2}$ corresponds to renormalization of the gaugino mass. When supersymmetry is unbroken, the former must be all the same and the latter must be zero, as stated in (2·7).

The divergent parts in $\tilde{C}_a$ (2·14) clearly represent a change in the beta function of the gauge fields, while the supersymmetry breaking finite parts represent different thresholds for these fields. To be more precise, we find that the contribution to the beta function from the hidden sector fields is given by

$$\varDelta b = -(2\pi)^4 c\,. \tag{3·3}$$

In other words, $b_{high} = b_{low} + \varDelta b$, where the subscripts denote the effective theories above and below the scale M of the hidden sector. Note that $\varDelta b$ is always negative; this is expected since the contribution to the beta function from charged matter is always negative.*)

So far we have discussed the simpler case of a single $U(1)$ gauge group here, in the case of the actual MSSM one has to consider the separate $SU(3)$, $SU(2)$ and $U(1)$ gauge groups. We will label the gauge groups by $r = 3, 2, 1$, respectively. If we want the gauge couplings to unify, then the value of $c^{(r)} = c$ must be independent

*) As mentioned in the introduction, our formalism does not include the case of "gauge messenger" models where the SM gauge group is nontrivially embedded into a larger group such as $SU(5)_{GUT}$. In such models, the contribution of the hidden sector to the beta function can have either sign.

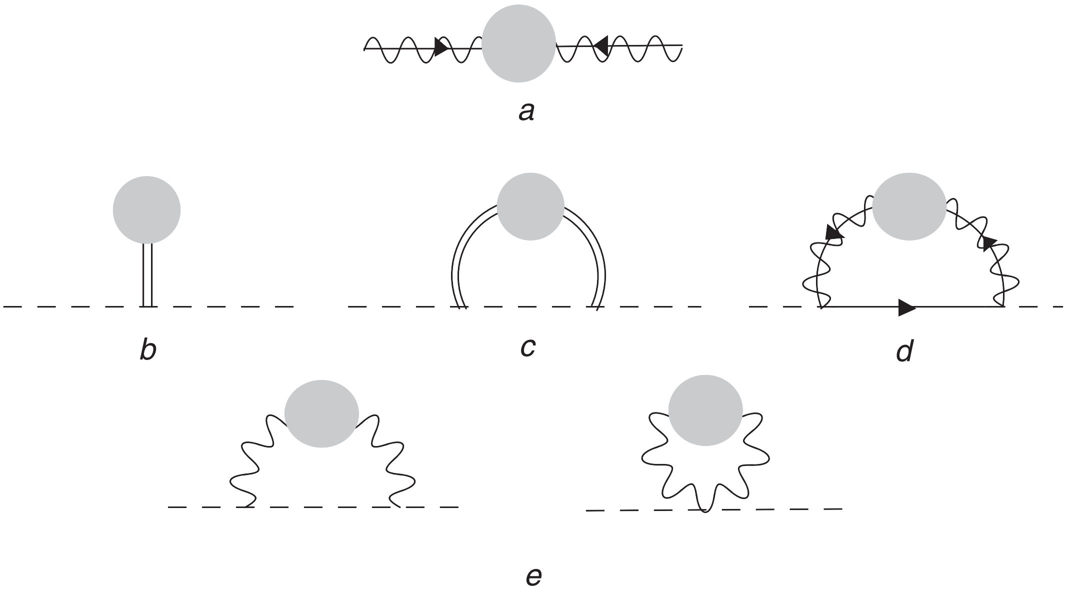

Fig. 1. The graphical description of the contributions of the two point functions to the soft masses. (a) represents the gaugino mass contribution from $\langle j_\alpha j_\beta \rangle$. In (b)–(e) the various contributions to the soft scalar masses are given: (b) $\langle J \rangle$, (c) $\langle JJ \rangle$, (d) $\langle j_\alpha \bar{j}_{\dot{\alpha}} \rangle$, and (e) $\langle j_\mu j_\nu \rangle$. It should be stressed that the blobs in the figures represent hidden sector correlation functions. The leading contribution in theories with messengers arises from one loop of the messengers, but in general when there are no messengers, it is more complicated.

of r (assuming $SU(5)$ normalization of the $U(1)$ factor of course) and we want the thresholds $\tilde{C}_a^{(r)}(0)$ to depend weakly on r. Moreover, if we want perturbative unification, then there is an upper bound on the magnitude of c. These are examples of some completely general constraints on the SUSY breaking sector that can be derived using our formalism.

Now, it is straightforward to find the sfermion and gaugino masses of the MSSM. In Fig. 1 we show the diagrams involving the current correlation functions which are responsible for the MSSM soft masses.

The gaugino masses arise at tree level in the effective theory (3·2); to leading order they are given by

$$M_r = g_r^2 M \tilde{B}_{1/2}^{(r)}(0)\,. \tag{3·4}$$

(Starting from this equation we use the hypercharge normalization of g_1 which differs from the GUT normalization by $\sqrt{5/3}$.)

We now compute the sfermion masses. When $\langle J \rangle$ is nonzero we get a tree level contribution to the sfermion $\tilde{f}$ (the superpartner of the fermion f) mass squared of the form $g_1^2 Y_f \zeta$ where Y_f is the $U(1)$ hypercharge of the sfermion. A more interesting effect arises at one loop. As we will soon see, the typical momentum in the loop is of order M, and therefore the low momentum effective Lagrangian (3·2) cannot be used. Instead, we use the full momentum dependence in the correlators (2·12) in three different one-loop diagrams, one with an intermediate D, one with an intermediate λ and one with an intermediate V. We easily find

$$m_{\tilde{f}}^2 = g_1^2 Y_f \zeta + \sum_{r=1}^{3} g_r^4 \, c_2(f;r) A_r \,, \tag{3·5}$$

where $c_2(f;r)$ is the quadratic Casimir of the representation of f under the r gauge group; and

$$\begin{aligned} A_r &\equiv -\int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \Big(3\tilde{C}_1^{(r)}(p^2/M^2) - 4\tilde{C}_{1/2}^{(r)}(p^2/M^2) + \tilde{C}_0^{(r)}(p^2/M^2) \Big) \\ &= -\frac{M^2}{16\pi^2} \int dy \left(3\tilde{C}_1^{(r)}(y) - 4\tilde{C}_{1/2}^{(r)}(y) + \tilde{C}_0^{(r)}(y) \right) . \end{aligned} \tag{3·6}$$

As stated above, the typical momentum in (3·5) is of order M rather than zero.

Although we did not prove it in general, the integrals in (3·6) should be UV convergent. Otherwise we would need a counter term for the sfermion masses which cannot be present in a theory with spontaneously broken supersymmetry.

We make two comments about these results. First, it is clear from this formalism that the gaugino masses are not a priori related to the sfermion masses, nor are they necessarily related to the change in the beta function (3·3). Thus, there is no a priori reason why one cannot have gauge coupling unification without gaugino mass unification in general models of gauge mediation. Second, we see from (3·5) the well-known fact that an effective FI term ζ can be quite dangerous for gauge mediation, since it leads to a non-positive definite (i.e. proportional to hypercharge) contribution to the sfermion masses at lower-order in the gauge couplings. Thus to avoid tachyonic slepton masses, usually it is assumed that some symmetry forbids ζ (see e.g. the “messenger parity” of Ref. 21)). In our general formalism, we can characterize this symmetry quite simply as an invariance of the hidden sector under a $\boldsymbol{Z}_2$ symmetry which acts on J as $J \to -J$.

§4. Mass relations

4.1. *Relations at the scale M*

In the previous section, we have seen how all the MSSM sfermion masses are completely determined in terms of four real numbers (ζ, A_1, A_2, A_3) which are derived from correlation functions in the SUSY-breaking sector. In this section, we analyze how this general result constrains the MSSM spectrum and leads to definite relations among the sfermion masses. We first consider the commonly assumed case $\zeta = 0$. Then there must be two relations amongst the sfermion soft masses which are valid in general. These mass relations can be easily derived by using the facts that each generation of the MSSM is separately anomaly free in $U(1)_Y$, and the mixed $U(1)_{B-L}$ – gauge anomalies also vanish. From the general form of the sfermion masses (3·5), it follows that

$$\mathrm{Tr}\, Y m^2 = \mathrm{Tr}\, (B-L) m^2 = 0 \,, \tag{4·1}$$

where the trace is over the MSSM sfermions in a given generation, and Y and $B-L$ stand for the hypercharge and $U(1)_{B-L}$ quantum numbers of the given sfermion, respectively. More explicitly, the mass relations are given by

$$\begin{aligned} &m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 = 0\,, \\ &2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2 = 0\,. \end{aligned} \tag{4·2}$$

These relations have been derived before in the context of various specific SUSY-breaking models (see e.g. Refs. 22)–26)). More recently, they have been discussed in Ref. 19) in the context of models with strong hidden sector renormalization effects. However, as our discussion makes clear, these relations are completely general features of gauge mediation, which do not depend on any specific form of the hidden or messenger sector (indeed, there need not even be any invariant distinction between the two). Thus, these relations offer a completely model independent test of gauge mediation which could in principle be carried out at the LHC or the ILC. Moreover, these sum rules could in principle be used to distinguish gauge mediation from other popular mediation schemes. In particular, the $B-L$ mass relation is violated in mSUGRA and various modifications of anomaly mediation which fix the slepton mass problem.

Next let us discuss the case that $\zeta \neq 0$. Then there should only be one relation amongst the sfermion soft masses. Indeed, it follows immediately from (3·5) that

$$\begin{aligned} &\mathrm{Tr}\,Y m^2 - g_1^2 \zeta \mathrm{Tr} Y^2 = 0\,, \\ &\mathrm{Tr}\,(B-L) m^2 - g_1^2 \zeta \mathrm{Tr}\,(B-L) Y = 0\,, \end{aligned} \tag{4·3}$$

which means that the linear combination $\mathrm{Tr}\,Y m^2 - \frac{5}{4}\mathrm{Tr}\,(B-L)m^2 = 0$ defines the one surviving mass relation. Explicitly, this takes the form

$$6m_Q^2 - 9m_D^2 + 3m_U^2 - 6m_L^2 + m_E^2 = 0. \tag{4·4}$$

Again, we emphasize that this sum rule is a completely model independent prediction of gauge mediation.

Finally, let us point out that there would be in principle two more relations relating the Higgs soft masses to the each other and the sfermion masses. However, since the Higgs soft masses generally pick up an additional contribution from whatever effect which generates μ and $B\mu$ (see e.g. §5), we expect that these additional mass relations will in general not be robust predictions of general gauge mediation. The same statement might apply also to the third generation mass relations, since the top Yukawa is large. On the other hand, since the two light generations couple to the Higgs fields very weakly, the precise details of the mechanism which generates μ and $B\mu$ hardly affect these mass relations.

4.2. *Corrections to the sum rules*

The sum rules derived in §4.1 hold at the characteristic mass scale M, which on general grounds must be at or above the electroweak scale (in models with messengers M can be thought of as the messenger scale). In general one must take into account the running of the soft masses in the MSSM in order to obtain low-energy spectrum, and this could potentially affect the sum rules. In fact, we will see that the sum rules for the first and second generation are quite robust under MSSM RG evolution. For simplicity, we will assume $\zeta = 0$ in this subsection.

Defining $S_Y^{(i)} = \mathrm{Tr}\, Y m_i^2$ and $S_{B-L}^{(i)} = \mathrm{Tr}\,(B-L) m_i^2$ as the sum rules for the ith generation and $S = \sum_i S_Y^{(i)} + m_{H_u}^2 - m_{H_d}^2$, it is straightforward to compute for the first two generations, using e.g. the formulas in Ref. 27), the one-loop running of $S_Y^{(i)}$ in the MSSM:

$$16\pi^2 \frac{dS_Y^{(i)}}{dt} = 2g_1^2 (\mathrm{Tr}\, Y^2) S \tag{4·5}$$

and

$$16\pi^2 \frac{dS_{B-L}^{(i)}}{dt} = 2g_1^2 (\mathrm{Tr}\, Y(B-L)) S\,, \tag{4·6}$$

where, again, the trace runs over just one sfermion generation. (For the third generation there are additional complications due to the Yukawa couplings and A terms.) In gauge mediation defined without any modification in the Higgs sector, $S = 0$ at M, and so these sum rules are preserved at all scales. However since we are allowing for potential modification to the Higgs sector in §5, we should keep in mind that there is in general an inhomogeneous correction piece $m_{H_u}^2 - m_{H_d}^2$ for both $S_Y^{(i)}$ and $S_{B-L}^{(i)}$. Fortunately, since these corrections are proportional to α_1, they are typically small for reasonable values of $m_{H_u}^2$ and $m_{H_d}^2$. Additionally there are also small corrections due to the MSSM D-terms after EWSB which can be found in Ref. 25) and are $\mathcal{O}(m_z^2)$.

§5. Comments on the $\mu/B\mu$ problem

One of the standard difficulties in gauge mediation models is the $\mu/B\mu$ problem: how to generate the couplings

$$B\mu H_u H_d + \int d^2\theta \mu H_u H_d + \text{c.c.} \tag{5·1}$$

with μ and B of the right order of magnitude. Let us try to address this in our very general framework.

Clearly, we need to couple the two Higgs fields $H_{u,d}$ to the hidden sector. One approach is to assume the existence of a hidden sector chiral operator $\mathcal{A}$ with coupling

$$\lambda \int d^2\theta \mathcal{A} H_u H_d \tag{5·2}$$

and vev

$$\lambda \langle \mathcal{A} \rangle = \mu + \theta^2 B\mu\,. \tag{5·3}$$

The operator $\mathcal{A}$ can be a fundamental field in the hidden sector theory. For example, this is the case in the NMSSM, if we view the singlet field of that model as a part of the hidden sector and identify it with $\mathcal{A}$. A problem with that is that unless certain discrete symmetries are imposed, a large tadpole for $\mathcal{A}$ is generated, leading to a need for fine tuning Ref. 28). Alternatively, $\mathcal{A}$ can be a hidden sector composite field whose short distance dimension is $\Delta > 1$. Its expectation value is naturally of order $M^{\Delta} + \theta^2 M^{\Delta+1}$. However, in this case the coupling λ in (5·2) is dimensionful and it is suppressed by a power of a large scale, e.g. $\lambda \sim 1/M_{Planck}^{\Delta-1}$. Therefore, the effect of the interaction (5·2) is negligible. Similar comments apply to other couplings like $\int d^4\theta \mathcal{A}^\dagger H_u H_d$.

The only kinds of couplings for which these comments do not apply are

$$\int d^2\theta(\lambda_u \mathcal{O}_u H_u + \lambda_d \mathcal{O}_d H_d)\,, \tag{5·4}$$

where $\mathcal{O}_{u,d}$ are composite operators with appropriate $SU(2)_L \times U(1)_Y$ quantum numbers and $\lambda_{u,d}$ are coupling constants. For example, such operators can originate at short distance from bilinears in the charged hidden sector fields. In this case the coupling constants $\lambda_{u,d}$ are dimensionless. Examples of such couplings appear in models with messengers (see e.g. Ref. 29)). In the coming discussion we will assume that the short distance dimension of $\mathcal{O}_{u,d}$ is two.

Above we defined gauge mediation as a situation in which in the limit $\alpha_{1,2,3} \to 0$ the theory decouples into a hidden sector and an MSSM sector. In the presence of the couplings (5·4) we must extend this definition to include the limit $\lambda_{u,d} \to 0$. Then, just as we have used perturbation theory in $\alpha_{1,2,3}$ to examine the effect of the hidden sector on the MSSM, we can also expand in $\lambda_{u,d}$. At leading order only couplings of the Higgs fields are affected. Using the hidden sector correlation functions

$$\langle \mathcal{O}_u \mathcal{O}_u^\dagger \rangle \qquad ; \qquad \langle \mathcal{O}_d \mathcal{O}_d^\dagger \rangle \qquad ; \qquad \langle \mathcal{O}_u \mathcal{O}_d \rangle\,, \tag{5·5}$$

where $\mathcal{O}_{u,d}$ stand for the full chiral superfields, we can generate Higgs masses, and the couplings (5·1). (In hidden sector models with multiple scales, we can also generate operators of the form (5·2), etc. with $\mathcal{A}$ given by a composite hidden sector operator. But in these cases the operator will be suppressed by a high scale in the hidden sector, not M_{Planck}.) Assuming that the correlation functions in (5·5) are given by powers of M, we naturally find $\mu \sim \lambda_u \lambda_d M$ and $B \sim M$. For $\lambda_u \lambda_d \sim \frac{\alpha}{4\pi}$ the generated μ is of the right order of magnitude, but B is too large. This is a well known problem with gauge mediation models (see e.g. Ref. 29)). In this general language, we see that the problem is clearly in the assumption that all the correlation functions are given by powers of M. One can certainly imagine that with the right hidden sector, some of the correlation functions in (5·5) are smaller than others, and this could lead to $B \lesssim \mu$ which are of the same order as the other soft breaking terms. For instance, this could conceivably arise either from anomalous dimensions in the hidden sector theory or from an approximate symmetry.*) The former possibility is not inconceivable, especially if the hidden

*) The use of anomalous dimensions for the $\mu/B\mu$ problem was recently discussed in Refs. 30) and 31). These models have two scales, with messengers being integrated out at the higher scale.

sector is strongly coupled, since the dimensions of the operators $\mathcal{O}_{u,d}$ (unlike the currents discussed above) are not necessarily protected by any symmetry. Finally, we point out that using higher point functions in the hidden sector, we can generate the effective dimension five and six operators of Ref. 32) thus potentially avoiding the little hierarchy problem.*)

It is clear that a much more detailed analysis of the correlation functions (5·5) is needed before we can conclude whether a typical hidden sector model can lead to a fully satisfactory solution of all phenomenological problems. We intend to return to such an analysis in the near future.

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Appendix A
—— *A Simple Example* ——

To illustrate the general techniques presented in the text, let us consider the simple case of minimal gauge mediation (for a $U(1)$ toy model), where

$$\delta\mathcal{L} = \int d^4\theta \left(\phi^\dagger e^{2gV}\phi + \tilde{\phi}^\dagger e^{-2gV}\tilde{\phi}\right) + \left(\int d^2\theta\, \lambda X\phi\tilde{\phi} + \text{c.c.}\right) \tag{A·1}$$

with $\langle X\rangle = M + \theta^2 F$. (Without loss of generality, we will take M and F to be real and set $\lambda = 1$.) Then we have complex scalar fields $\phi_\pm = (\phi \pm \tilde{\phi}^*)/\sqrt{2}$ with masses $m_\pm^2 = M^2 \pm F$; and two fermions ψ and $\tilde{\psi}$ which both have mass $m_0 = M$. The components of the current superfield are

$$\begin{aligned}
J(x) &= \phi^*\phi(x) - \tilde{\phi}^*\tilde{\phi}(x)\,,\\
j(x) &= -\sqrt{2}i(\phi^*\psi(x) - \tilde{\phi}^*\tilde{\psi}(x))\,,\\
\overline{j}(x) &= \sqrt{2}i(\phi\overline{\psi}(x) - \tilde{\phi}\overline{\tilde{\psi}}(x))\,,\\
j_\mu(x) &= i(\phi\partial_\mu\phi^*(x) - \phi^*\partial_\mu\phi(x) - \tilde{\phi}\partial_\mu\tilde{\phi}^*(x) + \tilde{\phi}^*\partial_\mu\tilde{\phi}(x))\\
&\quad +\psi\sigma_\mu\overline{\psi}(x) - \tilde{\psi}\sigma_\mu\overline{\tilde{\psi}}(x)\,.
\end{aligned} \tag{A·2}$$

*) These operators had been noticed and analyzed by various authors before Ref. 32) (see e.g. Ref. 33)). Our analysis here is in the spirit of Ref. 32) which considered a low energy effective Lagrangian obtained by integrating out generic short distance theories. Then these operators are the dominant ones in a systematic expansion.

From this, we obtain the correlators:

$$\begin{aligned}
\langle J(0)\rangle &= 0\,,\\
\langle J(x)J(0)\rangle &= 2D(x;m_+)D(x;m_-)\,,\\
\langle j_\alpha(x)\overline{j}_{\dot\alpha}(0)\rangle &= -2i(D(x;m_+)+D(x;m_-))\sigma^\mu_{\alpha\dot\alpha}\partial_\mu D(x;m_0)\,,\\
\langle j_\mu(x)j_\nu(0)\rangle &= 2\Bigg(\Big(\partial_\mu D(x;m_+)\partial_\nu D(x;m_+)-D(x;m_+)\partial_\mu\partial_\nu D(x;m_+)\Big)\\
&\qquad+\Big(\partial_\mu D(x;m_-)\partial_\nu D(x;m_-)-D(x;m_-)\partial_\mu\partial_\nu D(x;m_-)\Big)\\
&\qquad+2\eta_{\mu\nu}\Big(\partial^\rho D(x;m_0)\partial_\rho D(x;m_0)-m_0^2D(x;m_0)^2)\Big)\\
&\qquad-4\partial_\mu D(x;m_0)\partial_\nu D(x;m_0)\Bigg)\,,\\
\langle j_\alpha(x)j_\beta(0)\rangle &= -2(D(x;m_+)-D(x;m_-))\epsilon_{\alpha\beta}m_0D(x;m_0)\,,
\end{aligned}\tag{A·3}$$

where

$$D(x;m)=\int\frac{d^4p}{(2\pi)^4}\frac{ie^{ipx}}{p^2-m^2}\tag{A·4}$$

is the propagator for a scalar field with mass m. The expressions in (A·3) were derived by performing the free-field contractions on the correlators. Note that they are only valid for $x\neq 0$; for $x\to 0$ one must be more careful about including delta-function contact terms necessary for current conservation. These are most easily determined in momentum space, and we will take them into account below. As a check of these expressions, note that they satisfy the relations (2·7) in the SUSY limit $m_+=m_-=m_0$, with $C_a=2D(x;m_0)^2$.

From the correlators, we can extract the functions $\tilde C_a$ and $\tilde B_{1/2}$ by Fourier transforming the RHS of (A·3), substituting (A·4), and comparing with (2·12). For example, the first correlator of (A·3) yields

$$\begin{aligned}
\tilde C_0=\int d^4x\,e^{ipx}\langle J(x)J(0)\rangle &= \int d^4x\,e^{ipx}\Big(2D(x;m_+)D(x;m_-)\Big)\\
&=2\int\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2+m_+^2)((p+q)^2+m_-^2)}\,.
\end{aligned}\tag{A·5}$$

Performing similar manipulations for the other correlators, we obtain the final expressions

$$\begin{aligned}
\tilde C_0 &= 2\int\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2+m_+^2)((p+q)^2+m_-^2)}\,,\\
\tilde C_{1/2} &= -\frac{2}{p^2}\int\frac{d^4q}{(2\pi)^4}\left(\frac{1}{(p+q)^2+m_+^2}+\frac{1}{(p+q)^2+m_-^2}\right)\frac{p\cdot q}{q^2+m_0^2}\,,\\
\tilde C_1 &= -\frac{2}{3p^2}\int\frac{d^4q}{(2\pi)^4}\Bigg(\frac{(p+q)\cdot(p+2q)}{(q^2+m_+^2)((p+q)^2+m_+^2)}+(m_+\to m_-)
\end{aligned}$$

$$+\frac{4q\cdot(p+q)+8m_0^2}{(q^2+m_0^2)((p+q)^2+m_0^2)}-\frac{4}{q^2+m_+^2}-\frac{4}{q^2+m_-^2}\Bigg) \qquad (\mathrm{A{\cdot}6})$$

and

$$M\tilde{B}_{1/2}=2m_0\int\frac{d^4q}{(2\pi)^4}\left(\frac{1}{q^2+m_-^2}-\frac{1}{q^2+m_+^2}\right)\frac{1}{(p+q)^2+m_0^2}\,. \qquad (\mathrm{A{\cdot}7})$$

In $\tilde{C}_1$, we have included contributions from contact terms (the last two terms in the last line of (A·6)). These are required in order for $\langle j_\mu(p)j_\nu(-p)\rangle$ to satisfy the Ward identity as in (2·12).

At large p, the functions $\tilde{C}_a$ all have the form

$$\tilde{C}_a=\frac{1}{8\pi^2}\left(\log\frac{\Lambda^2}{p^2}+1\right)+\frac{1}{8\pi^2p^2}\left(m_-^2\log\frac{m_-^2}{p^2}+m_+^2\log\frac{m_+^2}{p^2}-m_-^2-m_+^2\right)$$
$$+\mathcal{O}(1/p^4,\,(\log p^2)/p^4)\,, \qquad (\mathrm{A{\cdot}8})$$

i.e. they all agree up to $\mathcal{O}(1/p^2)$ but not necessarily at $\mathcal{O}(1/p^4)$. Note that the agreement at $\mathcal{O}(1/p^2)$ depends on the fact that the messengers satisfy the supertrace relation, $m_-^2+m_+^2=2m_0^2$. In general, we expect that the functions $\tilde{C}_a$ should agree up to $\mathcal{O}(1/p^2)$ if supersymmetry is spontaneously broken. One important consequence of this is that the integral (3·6) for the sfermion masses is always UV finite, even though the individual terms contributing to it are not.

Finally, let us compare to the well-known formulas for the one-loop gaugino and two-loop sfermion masses of minimal gauge mediation. From (3·4) and (A·7), we find

$$M_\lambda=2g^2m_0\int\frac{d^4q}{(2\pi)^4}\left(\frac{1}{q^2+m_-^2}-\frac{1}{q^2+m_+^2}\right)\frac{1}{q^2+m_0^2}=\frac{\alpha}{4\pi}\frac{F}{M}\times 2g(x)\,, \qquad (\mathrm{A{\cdot}9})$$

where $x=F/M^2$ and

$$g(x)=\frac{(1-x)\log(1-x)+(1+x)\log(1+x)}{x^2}\,. \qquad (\mathrm{A{\cdot}10})$$

This agrees precisely with the answer in Ref. 25); the factor of two in (A·9) is the Dynkin index for the pair of messengers; more generally it would be $2Y^2$ where Y is the $U(1)$ charge.

Next we compare with the formula for the sfermion masses Ref. 25):

$$m_{\tilde{f}}^2=g^4(A_0+A_{1/2}+A_1)\,, \qquad (\mathrm{A{\cdot}11})$$

where

$$\begin{aligned}
A_0&=-2G_2(m_+,m_-)\,,\\
A_{1/2}&=4G_1(0)(G_0(m_+)+G_0(m_-)-2G_0(m_0))+4G_2(m_+,m_0)+4G_2(m_-,m_0)\\
&\quad+4(m_+^2-m_0^2)G_3(m_+,m_0)+4(m_-^2-m_0^2)G_3(m_-,m_0)\,,\\
A_1&=-4G_1(0)(G_0(m_+)+G_0(m_-)-2G_0(m_0))-G_2(m_+,m_+)-G_2(m_-,m_-)\\
&\quad-4G_2(m_0,m_0)-4m_+^2G_3(m_+,m_+)-4m_-^2G_2(m_-,m_-)+8m_0^2G_3(m_0,m_0)
\end{aligned} \qquad (\mathrm{A{\cdot}12})$$

and

$$G_0(m) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + m^2} \,,$$
$$G_1(m) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 + m^2)^2} \,,$$
$$G_2(m_1, m_2) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 + m_1^2)((p+q)^2 + m_2^2)} \,,$$
$$G_3(m_1, m_2) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 + m_1^2)((p+q)^2 + m_2^2)} \,. \qquad (\text{A}\cdot 13)$$

(Despite appearances, these functions are symmetric under interchange of m_1 and m_2.) Comparing with (3·5)and (A·6), we find that the individual A_a agree precisely with the contributions from $\tilde{C}_a$, respectively. Note that the contact terms included in (A·6) were crucial for the agreement between A_1 and the contribution from $\tilde{C}_1$.

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