Exporters’ Exposures to Currencies: Beyond the Loglinear Model*

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Abstract
We extend the constant-elasticity regression that is the default choice when equities’ exposure to currencies is estimated. In a proper real-option-style model for the exporters’ equity exposure to the foreign exchange rate, we argue, the convexity of the relationship implies that the elasticity should depend on the exchange rate level. For instance, it should shrink to zero when the option to export becomes worthless, and that should happen at a critical exchange rate that is still strictly positive. We propose a class of tractable multi-regime regression models featuring, in line with the real-options logic, smooth transitions and within-regime dynamics in the foreign exchange exposure. We then analyze the exchange rate exposure of Chinese exporting firms and find that the model in which the moneyness of the export option has a positive impact on the exchange rate exposure detects a significantly positive and convex exposure for 40% and 65% of the firms depending on whether the market return is included in the regression or not.

JEL classification: C13, C22, G11

1. Introduction
It seems self-evident that exporters benefit from an appreciation of the currency of their export markets: if they are price takers abroad, their profit margins swell, and if they have some market power they can further improve the results by optimally lowering the foreign sales price. Thus, in a regression of stock returns on percentage changes on the exchange rate,1 the slope coefficient should be positive. By a similar argument, importers should be characterized by negative exposures.

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1 Throughout the article, the term exchange rate refers to the value of the foreign currency in units of home currency.
Yet, as we document in Section 3, it has been surprisingly difficult to demonstrate non-
zero currency exposures for individual stocks, especially if one tries to link the sign of the
exposure to, e.g., the firm’s net trade. Estimated exposures also seem quite unstable across
subsamples (Jorion, 1990). Part of the problem may be that the standard loglinear model is
inadequate. Many propose a convex relation and provide evidence that is in line with that
conjecture. Priestley and Odegaard (2007), for instance, conclude that squared exchange-
rate changes matter too, empirically, and Bartram and Bodnar (2012) similarly find that
the relation between stock returns and exchange-rate changes depends on the direction of
the exchange-rate change. We propose and test a model that might explain all these
phenomena.

To open the debate, we observe that it is not always made clear what form of convexity
one has in mind. If firm $j$’s market value is loglinear in the exchange rate $S$, apart from a
random-walk part $Z_j$, i.e., when $V_j = k_jS^{c_j}Z_j$, then for an exporter we expect a positive $c$;
and if $c$ exceeds unity, $V_j$ is convex in $S$. Priestley and Odegaard (2007) and Bartram and
Bodnar (2012) implicitly deem this to be insufficient and want the elasticity itself to be state-
contingent. We agree, but we also provide a model that explains why, and in what way, con-
vexity between $V$ and $S$ might lead to nonlinearities in the relation between stock returns and
relative exchange rate changes. We then test our model on a sample of committed Chinese ex-
porters, and we find wide support for convexity. Below we amplify our views on convexity,
which are based on real options theory, and our conclusions for testable equations.

1.1 Real Options: from Convexity in Value . . .

We start from a consensus opinion in this field: for given levels of volatility, competition,
and production capacity, we expect a convex relationship between the firm’s value and the
real exchange rate. In cash flow terms, a linear relation would hold only if both the foreign
price and production plan remain unaltered regardless of the exchange rate, but that does
rarely make sense. Because of the firm’s “real” option to adjust pricing and production, the
impact of adverse movements can be partially mitigated, and beneficial changes can be ex-
ploited further. Thus, under the above stylized (short run) conditions, cash flow is convex
in the exchange rate, a familiar result.

One may rightly object that the above cash-flow picture ignores lump-sum adjustment
costs. Especially at the export side, abandoning or re-entering a market is likely to require
one-off outlays that are largely irreversible. Lump-sum items do destroy the smoothness of
the cash-flow function. But, still in the short-term scenario, those discontinuities disappear
again if we consider not the future cash flows themselves but their present values. In terms
of market value, the metric studied in most empirical work, optimal dynamic management
of the option to totally liquidate or otherwise abandon the export business implies that, for
lower and lower exchange rates, the value of the export activities smoothly converges to-
ward its liquidation value, or to the value of the firm as a purely domestic entity if this is
the better alternative.2

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2 When the exchange rate approaches the critical level where the firm would optimally switch its
operating mode, the likelihood that the fixed adjustment cost will actually occur will smoothly ap-
proach the certainty level, so the stock price smoothly adjusts. Smooth pasting as a characteristic
of optimization was introduced by Merton (1973). Dixit (1993) offers an excellent discussion. Sercu
(1992) apply it to the value of an exporting firm, with a “mothballing” mode between the active-
plant mode and the liquidation mode.
1.2 To the Modeling of Returns

Note that if we are considering values not cash flows, one needs to distinguish between utterly giving up on export activities and temporarily suspending actual exports. For a given technology the former happens when the option to export is worthless, that is, the prospects of ever exporting, or exporting again, have become utterly unlikely. This is different from suspending actual exports, just like postponing the exercise of an American-style is different from the option having zero value. Thus, at constant technology and plant size, we expect the sensitivity of an exporter’s stock price to the exchange rate to approach zero when the latter attains very unfavorable levels. In addition, while that “very unfavorable” level may be quite low, it is implausible that the rate would have to fall all the way to zero (a level never attained).

To sum up, in value terms we should still see smooth convex relations even if exercising the real options requires irreversible lump-sum investments, and the derivative should drop to zero at a finite and strictly positive level of the real exchange rate \( S \). This second aspect is missing in a homogenous model like the loglinear one: with positive \( \gamma \), exposure in the sense of \( \frac{\partial V_j}{\partial S} \) drops to zero only if \( S \) approaches zero, which in turn can occur only if the consensus is that \( S \) will be stuck at zero forever. While it is conceivable that the critical rate at which the option to ever export may be so outlandish that one loses little by setting it equal to zero, the test equation should at least leave that choice to the data. This desirable feature is missing in the standard iso-elastic model, whose derivative is zero if and only if \( S = 0 \), i.e., never, and whose elasticity is constant even at zero.

The nonlinear regressions between returns, as run by Priestley and Odegaard (2007) or Bartram and Bodnar (2012), do not easily fit into our view either. What is estimated in those papers is actually an elasticity, or a left- and right-elasticity, but what is missing is a discussion why and how there would be convexity between logs of \( V \) and \( S \) rather than their raw levels. More fundamentally, if \( \frac{\partial V}{\partial S} \) must drop to zero when the rate falls sufficient far, the derivative must clearly depend on the level of the exchange rate. But in Priestley and Odegaard (2007) or Bartram and Bodnar (2012), the coefficients depend on the size and/or sign of the percentage change, not on the level. As such, these regressions can document only some average difference between the sensitivity to finite exchange-rate changes leftward versus rightward, ignoring the feature that neither of these sensitivities is probably constant. While their empirical success shows their intuition does capture a relevant feature of the data, in our view a more structured approach would add value.

Before we proceed, let us point out one appealing implication of this real-option view. It is known that the real exchange rate typically exhibits very little mean-reversion. If, once it arrives at a particular level, the rate stays in that neighborhood for a rather long term and if every neighborhood has its own exposure, then if one estimates exposures from different subperiods one could observe very different results, as found in the literature.\(^3\)

Note that considering values instead of cash flows does not entirely solve the problem of lump-sum switch costs. If there are fixed costs associated with suspending and re-entering a market, the value function still depends on whether one is actively trading or not. We return to this “hysteresis” issue below.

\(^3\)This would be even more the case if one considers big firms, like Jorion’s multinationals: these usually have both exporting and importing divisions beside foreign subsidiaries whose exposures may have any sign. As a result the entire value, being the sum of positively and negatively sloped convex functions, might even be U-shaped in \( S \).
1.3 Other Issues in Exposure Modeling

Two elements could complicate the view that was just proposed. First, the above story is about an unhedged firm. But many firms do hedge their transaction exposures and their expected cash flows over a chosen horizon. In the mythical case of a linear relation between cash flow $C$ and exchange rate $S$, all exposures within the hedging horizon can be eliminated by a forward hedge. But if the relation is nonlinear, as we just argued it should be, a static linear hedge is insufficient. Only if the firm hedges its entire value, if the exchange rate’s path is smooth, and if there is continuous dynamic updating, can the relation between value and exchange rate disappear altogether. While few firms are that ambitious, it is unclear how far they actually do go, so the realized relation may be a sequence of ever-shifting convex shapes that defies easy modeling.

A similar problem arises with operational hedging. Operational hedging is akin to being ambidextrous, i.e., being an exporter and importer at the same time. In a static model, this may lead to a U-shaped total-value function $V(S)$, but in practice that relation would be unstable. This is especially likely for multinationals, like in Jorion (1990), that continuously change shape as divisions are added, expanded, shrunk, or sold. It is with these two problems in mind that the sample in this version of the paper has been selected: the Chinese exporting firms we study are quite unlike the ambidextrous US multinationals and do not have access to financial hedges.4

But even for that sample problems remain. The potentially most bothersome complication, in general, is that the setting is not constant. Volatilities do change over time; flourishing export activities should attract competition; and, especially, the firm is likely to modify its installed capacity depending on the current state and prospects of the export- or import-related activities. It could be argued that expanding or shrinking the production apparatus is just another example of lump-sum adjustment costs, which should be anticipated by the market and therefore smoothed out in the valuation function. Likewise, if competitors enter or leave the markets, that can still be foreseen and anticipated in the valuation process. But that textbook view assumes the standard rational-expectations setting where all possible future events are perfectly known and their implications perfectly understood by everybody. In reality, there are unknown as investors’ understanding of the world is far from complete. If markets think just a few years ahead rather than perfectly anticipating all possible events from here to eternity, valuations could be more characterized by a sequence of short-term models. Instead of having a single convex relation, we might face a mixture of several relations. If so, convexity cannot be expected to hold unconditionally and everywhere.

In practical applications, one should also deal with another issue on which scholars’ opinions seem to vary: do we need a market-return regressor too, beside the exchange-rate variable? Conceptually, a hedger would add the market return only if hedging relies on both market-index and currency futures or forwards. In contrast, an academic interested in exposure *per se* may feel tempted to include the market to control for many other factor and obtain more precise estimates. But the risk is that, if firms in a market tend to have similar exposures, the market return will pick up a substantial part of the currency effect. Since the firm’s exposure then reflects only exposure over and above the exposure picked up via the market (the Frisch-Waugh Theorem), the risk is that what we find is just leftovers, hard to model and to make sense of. This risk is easily illustrated for our sample of Chinese exporters. Both our firms and our market index exhibit strong exposures.

4 We thank the anonymous referee for the recommendation to focus on this sample.
As a result, while in the simple regression of returns on exchange-rate changes, 70 of our 102 firms have positive significant exposures, this number drops to 26 when the market return is added.

In light of all this, our research questions are (i) do we see evidence of non-constant exposures as predicted by the real-option literature? (ii) is the world sufficiently simple so that we can still expect exposures to fall at lower and lower rates? (iii) does value seem to be convex in the real exchange rate elsewhere? and (iv) to what extent is the evidence robust to the market return being added or omitted as a regressor?


One antecedent of the standard loglinear regression is the Johnson (1960) and Stein (1961) minimum-variance hedge problem: if a cash flow or asset worth $y$ is to be hedged using an instrument with value $x$, then the regression coefficient $b$ in $y = a + bx + e$ is the hedge ratio that minimizes the residual variance of the value to be hedged $y$. Dumas (1978), reiterated in Adler and Dumas (1984), imported this concept into international finance, choosing the stock’s future value and the future exchange rate as the $y$ and $x$ variable, respectively, and naming this regression coefficient $y$’s exposure to the exchange rate.

If time series data are used and both $y$ and $x$ are nonstationary, or close to, then the regression has to work with first-differenced numbers, at the very least. Actually, to reduce heteroscedasticity issues, the preferred version is to work with percentage changes rather than dollar changes, implying that the regression coefficient delivers an elasticity rather than the partial-derivative-like number (like a hedge ratio). If excess returns are used, the regression is

$$\left(\tilde{r}_j - r_d\right) = x_j + \gamma_j(\tilde{s} + r_f - r_d) + e_j,$$

where $\tilde{r}_j$ denotes the return on a risky asset $j$; $r_d$ ($r_f$) is the domestic (foreign) risk-free rate; and $\tilde{s}$ the simple percentage change in the exchange rate, typically expressed in home currency units. In most cases, the risk-free rates are omitted without any noticeable harm. In doing so, the exposure literature followed standard practice in the Capital Asset Pricing Model (CAPM)-inspired market-model regression.

A more direct link with asset pricing theory emerged when Sercu (1980), generalizing Solnik (1974)’s International CAPM, realized that Dumas’ exposures, in their multivariate guise, could be used to develop a quasi one-factor CAPM for stocks hedged against exchange risk. Adler and Dumas (1983) estimate the equation. In an alternative version, Sercu (1980) merges the CAPM for bonds with that for hedged stocks into a standard multifactor CAPM-cum-regression, where the market portfolio shows up next to the exchange rate changes of all countries. In the two-country case, for instance, the model is

$$E(\tilde{r}_j - r_d) = \beta'_j E(\tilde{r}_w - r_d) + \gamma'_j E(s + r_f - r_d),$$

5 One drawback of our Chinese sample is that since all these firms are known to be active exporters, a zero-value point for the option to export cannot logically be observed in the data. In another sample, one could test whether in that point the limiting value of the exposures is zero or not.
where $\tilde{r}_w$ denotes the return on the world market $w$ and where $\beta_j'$ and $\gamma_j'$ are the slope coefficients from

$$\tilde{r}_j = \beta_j'\tilde{r}_w - \text{r}_d + \gamma_j'(\tilde{s} + \text{r}_f - \text{r}_d) + \epsilon_j.$$  

(3)

In multi-country models, each and every exchange rate’s percentage change shows up in the CAPM and in its associated regression as a separate regressor.

Since Jorion (1990), these separate, bilateral exchange-rate changes are often replaced by a single return on a bucket of currencies, typically trade-weighted. Again, in most applications the risk-free rates are omitted from the regression; in practice, finally, the world market is often replaced by stock $j$’s home-country market index return, denoted $\tilde{r}_m$. The motivation is to reconcile the Dumas regression with the national market model, often still seen as the premier return-generating model, and to obtain more precision by controlling for a host of factors that affect all stocks in similar ways. The basic Jorion-type regression, in short, is

$$\tilde{r}_j = \beta_j\tilde{r}_m + \gamma_j\tilde{s} + \epsilon_j,$$  

(4)

with $\tilde{s}$ denoting the percentage price change for the basket, an average of the currency-by-currency percentage price changes. The Jorion (1990) model, or closely related variants, has been the workhorse in this literature. 6

Most of the subsequent literature has worked with variants of this regression. Much of the tinkering was motivated by the, in a way, disappointing results for the model: with US data, surprisingly few companies came up with significant exposures (see e.g., Jorion, 1990, 1991; Bodnar and Gentry, 1993; Amihud, 1994; Choi and Prasad, 1995). Non-US data, from supposedly more open countries, provided mixed results: He and Ng (1998) report one stock out of four to be exposed in Japan, and Kiymaz (2003) one out of two for Turkey, but others found only weak evidence of currency exposure for Australia (Khoo, 1994) or broad samples of countries including Japan (Griffin and Stulz, 2001; Dominguez and Tesar, 2006). Chue and Cook (2008) estimate the exposure of emerging countries and find that the depreciation of domestic currencies has a negative impact on stock returns.

All of these studies look at stocks as priced in their home stock market. Translated asset prices, however, in the sense of assets where most or all the price-setting happens in the foreign market, are clearly exposed; see e.g., Jorion’s results for foreign multinationals listed in the USA, or Adler, Dumas, and Simon (1986)’s findings for dollar returns on foreign market indices. Thus, the picture seems to be disconcertingly close to Solnik (1974)’s original CAPM, where each stock has a clear nationality in the sense that its value, when expressed in its home currency, moves independently of exchange rates against that home currency.

This may reflect systematic and successful covering against exchange risk. Multinationals can do so via long-run operational hedges (Logue, 1995; Bartram, Brown, and Minton, 2010) or via financial hedges. To test this, exposures $\gamma$ have been modeled as linear in variables that probably correlate with the firm’s desire to hedge, like the firm’s

foreign sales or operations as a fraction of the total, the fraction of exports and imports, its leverage, its liquidity, and so on. There is some evidence that firms that have higher operating exposures or are more vulnerable do hedge more, resulting in a lower residual exposure; but the effect is not strong (Allayannis, 1996; De Jong, Litgerink, and Macrae, 2000; Gao, 2000; Carter, Pantzalis, and Simkins, 2001; Pantzalis, 2001; Pantzalis, Simkins, and Paul, 2001; Williamson, 2001; Crabb, 2002). A related literature relates the exposures to firm characteristics, sometimes with some success (Bodnar and Gentry, 1993; Williamson, 2001) and sometimes without much useful insights (Amihud, 1994; Fraser and Pantzalis, 2004).

Exposure may also be unstable. Many authors run subperiod regressions (Jorion, 1990; Amihud, 1994; He and Ng, 1998; Glaum, Brunner, and Himmel, 2000; Williamson, 2001; Doukas, Hall, and Lang, 2003), or work with rolling or moving data windows (Glaum, Brunner, and Himmel, 2000; Entorf and Jamin, 2002, 2007), or relate changes in exposure to the shifting weight of exports and imports (Allayannis, 1996). Another type of instability has been documented too, where exposure is changing depending on, e.g., the direction of the exchange-rate change (Priestley and Odegaard, 2007; Bartram and Bodnar, 2012). These non-linear models are different from the model proposed here in that we let the exposure depend on the level of the exchange rate, not on the size or sign of the percentage change. In our return model, exposure primarily depends on how far the exchange rate is from a critical level, and that parameter is not quickly adjusting to recent events in the currency market.

Regarding the data type, there may be an “intervalling effect” due to frictions in the trading process delaying the adjustment of the stock’s price to changes in the foreign exchange. Some report better results when using daily data rather than the usual monthly (Chamberlain, Howe, and Popper, 1997; Di Iorio and Faff, 2000; Glaum, Brunner, and Himmel, 2000), while others advocate longer holding periods (Bartov and Bodnar, 1994; Chow, Lee, and Solt, 1997; Chow and Chen, 1998; Di Iorio and Faff, 2000; Dominguez and Tesar, 2001a; Griffin and Stulz, 2001; Muller and Verschoor, 2006a, 2006b, 2006c). Even adding lagged exchange-rate changes has been considered (Bartov and Bodnar, 1994; Allayannis, 1996) with mixed results.

Others have tweaked the input data. Regarding the market return, Bodnar and Wong (2003) experiment with equal versus value weighting. Others replace the market by a number of macro-variables (Gao, 2000), return determinants (Chow and Chen, 1998) or Fama–French factors (Doukas, Hall, and Lang, 1999). A branch of research has worked with orthogonalized regressors, either constructing in-sample-hedged \( \hat{r}_m \) variates like Allayannis (1996), Bartram and Bodnar (2007), Bodnar and Wong (2003), Bris, Koskinen, and Pons (2004), Entorf, Moebert, and Sonderhof (2011), Griffin and Stulz (2001), Kiymaz (2003), Priestley and Odegaard (2007), and Pritamani, Shome, and Singal (2004), or market-corrected \( \hat{\delta} \) data (Jorion, 1990; Choi and Prasad, 1995; Glaum, Brunner, and Himmel, 2000; Hagelin and Pramborg, 2002). The effect of using this particular type of constructed variables is that the coefficient of the uncorrected regressor becomes equal to the simple regression slope, while the coefficient of the orthogonalized regressor remains equal to the multiple-regression slope (the Frisch–Waugh theorem). Undesirably, however, the regular t-statistics no longer apply, so this is not a recommendable procedure (Liu, Sercu, and Vandebroek, 2015).

Much attention has also been devoted to the exchange-rate variable, especially to the use of a fixed basket à la Jorion. The objective is to avoid multicollinearity between...
the various exchange rate changes, which all share a common component (the reference-currency factor). There would be no loss of information if all firms have bilateral exposures to the various currencies that are proportional to the weights used in the basket. This is unlikely, of course: firms trade with different countries, chose their own selective hedging policies, face different demand elasticities depending on their industry, etc. So if the implicit assumption of proportional exposures is not met, the use of a basket comes at a cost: the regressor measures the ideal basket with error, inducing a standard attenuation-toward-zero bias. Dominguez and Tesar (2001a) find that using bilateral rates beats the currency basket. Similarly, some obtain better results when multiple, more focused buckets are used, constructed, e.g., via factor analysis (Miller and Reuer, 1998) or on a regional basis (Muller and Verschoor, 2006a, 2006b, 2006c). Khoo (1994) collects firm-by-firm trade data and builds a firm-specific basket. This is unusually painstaking, but still ignores other determinants of exposure, like demand elasticity and hedging policies. A last issue is whether one should not use real returns, and changes in the real exchange rate, rather than nominal units. In practice, using real data hardly affects the conclusions (Bodnar and Gentry, 1993). Nor do two-step procedures where the variates are first regressed on lagged variables with some predictive power, like lagged changes (Amihud, 1994) or macro-variables (Gao, 2000) to filter out the predictable part of changes.

A last point of debate has been the use of portfolio data versus individual stock data as the left-hand side variable. The attraction of aggregate data is that there is likely to be diversification, meaning a lower total variance and, it is hoped, especially a lower residual variance, implying more precise estimates. In addition, one can easily build balanced panels without creating a survivorship-biased sample, as one would have to do with individual stock data. The obvious risk is firm heterogeneity: if the component stocks have different exposures, and especially if their exposures differ in sign, the resulting aggregate may have too little exposure to be detectable. Many authors, on balance, come down against portfolios (Khoo, 1994; Choi and Prasad, 1995; Allayannis, 1996; Muller and Verschoor, 2006a, 2006b, 2006c). Yet the context of the test matters too. For a stand-alone exercise, individual stock data may be recommendable. But for CAPM tests, portfolio data are much more wieldy. Griffin and Stulz (2001), Dumas and Solnik (1995), and De Santis and Gerard (1998), for instance, work with country indices.

Some papers do go beyond the loglinear model, like when He and Ng (1998) and Dominguez and Tesar (2006) find that the direction of exposure depends on the exchange rate level or when Priestley and Odegaard (2007) and Bartram and Bodnar (2012) find a nonlinear relation with exchange rate exposure changing in terms of depreciation to appreciation periods. By and large, though, the empirical literature has not ventured far from the Jorion regression, even though theoretical work suggests much richer models. Our ambition is to reduce the gap between theory and empirics by presenting a model whose features are inspired by theory but whose mathematical form is amenable to empirical work.

3. Modeling the Time-Varying Elasticity of Firm Value to Exchange Rates

In this section, we derive a regression model to estimate the time-varying elasticity of the exporter’s firm value to the exchange rate. The assumption of constant technology and plant size, adopted in Section 3.1, is relaxed in Section 3.2.
3.1 A Static Real-Option Model for an Unhedged Exporter

We take a semi-parametric approach to modeling the dependence of the firm value on the real exchange rate. In this article we consider a firm that combines a low-exposure domestic activity with exports. Denote the level of the real exchange rate at which the value of the foreign exchange exposure of the exports division becomes zero by \( S_x \) and the corresponding alternative value of those assets by \( c_x \). At this stage the model is static in the sense that it takes the technology and the valuation equation as constant, implying that the critical rate \( S_x \) is constant too.

The functional form that links the firm’s value and the exchange rate should capture the smoothness characteristic suggested by the real options literature and should also allow a tractable transition to a test equation. To that end we approximate the value of the exposed entity \( h_x(S_t) \) by its second-order Taylor expansion to the right of the threshold point \( S_x \):

\[
V_t / C_s h_x(S_t) = h_x(S_x) + h_x(S_x) \frac{S_t - S_x}{C_s} + h''_x(S_x) \left( \frac{S_t - S_x}{C_s} \right)^2 + \ldots
\]

Note that, by construction, the proposed specification satisfies the so-called value matching condition from the real options literature (Dixit, 1993). According to this condition, the export-like activity is replaced by its alternative when its value falls to the level of the alternative value:

\[(\text{value-matching condition}) : h_x(S_x) = c_x. \]

If this condition fails, arbitrage profits would be possible.

At the critical exchange rate below which the firm becomes purely domestic, the value functions to the left and to the right should also have the same slope (smooth pasting). Almost by definition of being domestic, the valuation below that critical rate has low exposure. One testable simplifying assumption, in that light, is that the value of the fall-back option is completely unexposed. Smooth pasting then says that at \( S_x \) the slope to the right should be zero too:

\[(\text{zero-exposuresmooth-pastinghypothesis}) : h'_x(S_x) = \frac{\partial h_x(S)}{\partial S} = 0. \]

Finally, as mentioned also in the introduction, we expect that the value function is a convex function of the exchange rate. The reason is that firms have the flexibility to optimally adjust production in response to the exchange rate. For given levels of volatility, competition,
and production capacity, this flexibility implies that the revenue per unit of an exportable good (Sercu, 1992) or the entire cash flow from exporting (Franke, 1991; Sercu and Van Hulle, 1992) become convex functions of the exchange rate:

\[
(\text{convexity}): h''_x(S_x) \geq 0. \tag{8}
\]

To prepare the real-option model for the firm value in Equation (5) for estimation we derive the implication for stock returns. We first work out the change of the squared excess exchange rate:

\[
\left(\frac{S_t}{C_0} - \frac{S_{t-1}}{C_0}\right)^2 = 2\left(\frac{S_t}{C_0} - \frac{S_{t-1}}{C_0}\right)\Delta S_t,
\]

where \(\Delta S_t := S_t - S_{t-1}\) and \(S_{t-1,t} := S_{t-1} + (1/2)\Delta S_t = (S_{t-1} + S_t)/2\). Define, in addition, the indicator for positive time values,

\[
1_{t,x} := \begin{cases} 1 & \text{if } S_{t-1} > S_x, \\ 0 & \text{otherwise}. \end{cases} \tag{10}
\]

This allows us to write the value change of the exporter’s activities, \(\Delta V_t\). In what follows, we do not impose the zero fall-back exposure hypothesis \(h'_x(S_x) = 0\). For constant \(1_{t-1,x}\) the first difference then becomes

\[
\Delta V_t \approx \left[h'_x(S_x) + h''_x(S_x)2(S_{t-1,t} - S_x)\right]1_{t-1,x}\Delta S_t, \tag{11}
\]

which, given our quadratic approximation, yields a first derivative that is linear in the exchange rate.\(^{10}\)

Under the above assumptions, we thus obtain the following model for returns:

\[
\frac{\Delta V_t}{V_{t-1}} = \left[h'_x(S_x) + h''_x(S_x)2(S_{t-1,t} - S_x)\right]1_{t-1,x}\Delta S_t, \tag{12}
\]

with constant values for \(h'(S_x)\) and \(h''(S_x)\) in the short run (i.e., for given \(S_x\)). Finally, we can include a domestic market-model term as a simple way to account for domestic activities and general market evolutions (such as interest rates) which influence both the international activities and the domestic activities. It is an empirical matter whether, this way, too much of the firm’s exposure is picked via by the market regressor, leaving just unsystematic scraps for the currency regressors.

Including also an intercept \(x\) and an error term \(\epsilon_t\), all this leads to our short-term model for the firm’s currency exposure:

\[
r_t = x + \beta r_{mt} + S_{t-1} \left[h'_x(S_x) + h''_x(S_x)2(S_{t-1,t} - S_x)\right]1_{t-1,x} + \epsilon_t, \tag{13}
\]

with \(r_t = \Delta V_t/V_{t-1}\) the firm’s stock return and \(s_t = \Delta S_t/S_{t-1}\) the exchange rate return.

### 3.2 Toward a Long-Run Exposure Model for Stock Returns

Like the typical analytical framework in this literature, the above takes the firm’s plant size or technology as given, and likewise for interest rates and volatility. The smooth-pasting

\(^{10}\) A slightly different version holds when during the period \([t-1, t]\) the indicator \(1_x\) changes, but as our sample firms are all active exporters this is immaterial for our tests.
solution for a perpetual-lived contingent claim then implies constant critical values \( S_x \) and derivatives \( h_x \) and \( h''_x \). In practice, a firm threatened by adverse exchange-rate movements can make “long-run” adjustments by changing its technology or plant size, etc. From that perspective it would be preferable to adopt a time-varying critical rate: if the spot rate crosses that level, the option to trade loses its relevance as long as the technology remains unadjusted. Adjusting the technology is what many firms do, more or less continuously—witness for instance the come-backs of Japan’s exporters, time and again, despite currency setbacks.

3.2.a. A barrier model for investment decisions affecting the critical value

We follow the logic of investment models with an optimal-timing issue. When the firm’s competitive position deteriorates and the exchange rate drops below the break-even value, there is a certain value in delaying the required investment because any such investment is largely irreversible while the current depreciation of the foreign currency may soon be reversed. We model this through a threshold model in which the investment is made when the exchange rate crosses a critical barrier. Similar critical barriers arise in, e.g., structural credit risk models, where the option to default is exercised when asset value reaches a default barrier, or, in an application closer you ours, in optimal investment-timing models with stochastic interest rates (Ingersoll and Ross, 1992) where at constant cash flows the decision happens as soon as interest rates decrease to the acceptance interest rate, which is below the break-even rate. Dixit and Pindyck (1994) and Trigeorgis (1996) offer classic reviews.

In line with this, the firm invests as soon as the rate falls below a certain level. We do not regard that level as constant in absolute terms: competing firms can and do adjust too, all following their own optimal timing, and their joint action continuously changes the standards. If all firms need to stay close to the state-of-the-art technology, that technology standard and the resulting critical exchange rate are adjusting slowly when the exchange rate is drifting down. We approximate the result of all firms’ slowly implemented adjustments as tracking a moving average of lagged rates, at least when the moving average is drifting down. We approximate the result of all firms’ slowly implemented adjustments as follows: technology is adjusted so that the resulting critical rate remains at least \( B \) units below a moving average of past rates. Stated differently, when the moving average is drifting down too close to the critical rate implied by the current technological standard, investments are undertaken to lower the critical rate.\(^{11}\)

\[
S_{x,t} = \min(S_t - B, S_{x,t-1}), \quad \text{with} \quad S_t := \frac{\sum_{l=1}^{L} S_{t-l}}{L}. \tag{14}
\]

We initialized the variable as \( S_{x,0} = \max(S_0 - B, 0) \).

In considering how to set \( L \) we combine economic and statistical viewpoints. Statistically, the issue is the time series properties of the variable \( S_t - S_{x,t} \). Consider two extreme cases. First, in the presence of asymmetries, one undesirable limiting case is that the rate almost always changes in the favorable direction; if so, in the limit the critical value

\(^{11}\) This assumes that the required investments are always worthwhile. In a general situation this may not be true: firms can throw in the towel and quit exporting. But in our sample of firms that are continuously exporting, this obviously does not happen.
would be constant, in which case we have an obvious unit-root problem with the term $(S_{t-1} - S_{x,0})$ in the regression. In our data set this does not occur: the Yuan does appreciate most of the time, implying a negative drift for the foreign currency and relatively frequent updates in the critical rate that prevent an accumulation of past shocks. In the other limiting case, then, if there rate would almost always go down, there would be continuous updating of the critical level $S_{x,t}$. Then the variable to watch, $S_t - S_{x,t}$ would in principle be a stationary process but could still be very close to unit root in a finite sample if $L$ is large, thus spelling trouble for the inference (Roll and Yan, 2000). In the empirical application we therefore perform standard unit-root tests for increasing values of $L$, and use the first window size where a unit root is no longer rejected in $S_t - S_{x,t}$ as a pragmatic indicator of trouble coming. It turns out that if we set $L$ beyond 18, then $S_t - S_{x,t}$ is getting too close to being unit-root by conventional standards.

In terms of economics, the fact is that there are no easily available international data that allow us to set the speed of adjustment—or, here, the length of the window over which the moving average is calculated. But $L = 18$ would mean that the firm is deemed to be able to undo an adverse change in competitiveness in, on average, like 9 months, which seems to be on the short side. We would have preferred a slower response. From that perspective it is reassuring to find that the choice of $L$ is not crucial. In the empirical section we also report the model estimation results for $L = 9$ and even for $L = 36$, and obtain qualitatively similar results as for $L = 18$.

3.2.b. Modeling the value effects of investment decisions

The next issue is how to handle the value effects of adjustments in plant and technology. Any such change affects both the form of $h(S)$ and the critical rate $S_{x,t}$. Recognizing that the derivatives in Equation (13) are no longer constant we now write the valuation at the critical rate as $h_{x,t}(S_{x,t})$ instead of $h_{x}(S_{x})$. There is no obvious equation available for how these derivatives are changing in the long run. But the final model should preserve the main novelty of the short-run model (13) relative to the standard loglinear equation: the exchange-rate elasticity may have a constant component and also contains a term proportional to $S_{t-1} - S_{x,t-1}$. We include the new insight by approximating the time varying elasticity as a conditionally linear function in $S_{t-1} - S_{x,t-1}$.

More formally, in the first line below we extract from Equation (13) the implied elasticity dynamics, and in the next line we approximate it by a function involving two constants, $\lambda_x$ and $\delta_x$:

$$\frac{\partial \log V_t}{\partial \log S_t} = \frac{S_{t-1}}{V_{t-1}} \left[ \frac{b_{x,t-1}(S_{x,t-1}) + b''_{x,t-1}(S_{x,t-1})}{2(S_{t-1} - S_{x,t-1})} \right] 1_{t-1,x}$$

$$(15)$$

with $\lambda_x, \delta_x \geq 0$ necessary conditions to ensure that firm value is an increasing, convex function of the exchange rate. For our test equation to be literally correct, the derivatives, scaled by $V/S$ would have to be constant, which is unlikely; so $\lambda_x$ and $\delta_x$ are to be read as averages. On the positive side, an increase in the firm’s scale or technology should still affect both the derivatives and the value in the same direction, so that the scaled derivative should be less variable than the raw one. Another advantage is that, this way, the standard model is nested within our more general equation.
All this leads to our most general model for the firm’s currency exposure:

\[ M_0 : r_t = \alpha + \beta r_{m,t} + [\lambda_x + \delta_x 2(S_{t-1} - \Sigma_{t-1})] I_{t-1,x} s_t + \epsilon_t. \]  

(16)

As noted already, in our test equation the elasticity is not constant like in the default model; rather, its implied path is Equation (15). So in our equation the elasticity is modeled, in the short run, as a piecewise linear function with a zero level and a zero slope at extremely low real exchange rates, and rising for higher rates. In the long run, the above still holds as long as the exchange rate is measured in excess of the critical rate that corresponds to the then relevant technology; below, we refer to this distance as the firm’s moneyness, by analogy of the terminology adopted in the options literature.\(^{12}\) In the standard regressions like Dumas (1978) and Jorion (1990), moneyness plays no role as \( \delta_x = 0 \), and \( \lambda_x \) refers to the exposure of the firm rather than that of the firm’s domestic fall-back option. Priestley and Odegaard (2007)’s elasticity is linear, but in the contemporaneous monthly percentage change rather than in moneyness in the beginning of the period. It is as if the relation \( V(S) \) is approximated by a local quadratic approximation that is somehow always the same, regardless of the moneyness of the particular situation. Bartram and Bodnar (2012) is like a piecewise linear analog to the Priestly–Odegaard quadratic, and similarly makes no reference to the level of the exchange rate vis-a-vis a benchmark rate.

To close, we note that our model is, in principle, a two-regime one. The critical values of the exchange rate are those where, at current technology, the option to ever export again becomes worthless. Our current sample is such that we can a priori rule this out. In other samples, such a critical value still may be so extreme that, in practice, it is never reached; then the activity de facto is still a one-regime one, in the sample. Otherwise we have a two-regime model.

4. Empirical Application: The Case of Chinese Exporters

Based on the history of stock returns \( r_t \), real effective exchange rates \( S_t \) and its transformations (the exchange rate return \( s_t \), the \( L = 18 \) month moving average of \( S_t \) denoted \( \bar{S}_t \)), we now wish to examine the relative gains of the proposed real-option model for a sample of Chinese exporters. Sections 4.1 and 4.2 describe the sample and test equations. The empirical results are presented in Section 4.3.

4.1 The Sample

The data we work with refer to a set of 102 Chinese firms that are systematic exporters. This sample has several desirable features. First, for these firms the export-to-sale ratio is impressive, averaging 60% as we shall see, promising a strong currency effect. Second, that currency exposure should be unobscured by hedging. Currently, Mainland China still has no FX forward and futures markets where firms can hedge their currency exposure. Although the offshore non-deliverable forward market has developed rapidly, it is open only to non-domestic players, and explicitly not to Chinese firms, not even for hedging. The choice of the sample also simplifies the estimation, because we know a priori that (i) the firms’ exposures are positive not negative, so we need just the export model, not a mixed import/export model; and (ii) all observations must be from the same regime.

12 The moneyness of a call, for instance, is the gap between the underlying and the strike price, albeit typically measured as a difference in logs rather than levels.
the positive-option-value one. Finally, if all firms are active exporters all the time we face less of a hysteresis problem. Indeed, as entering a foreign market requires substantial irreversible investments, there must be an inactivity zone of exchange rates where a firm that has not made the investment previously would stay out of the export activity, while a previously active exporter would continue to export (footnote 2). This implies that there would be two valuation rules depending on whether $S$ enters the zone from below or from above; in addition, the zone itself would have to be identified too.

Returns cover the period from July 2005 (the date the Central Bank stopped pegging to the US Dollar at 8.2777) until December 2013. We select the firms that have foreign sales exceeding 10% of total revenues for each of the years 2006–12. On the basis of the annual financial statements, there are 102 firms consistently having sufficient foreign sales, and the ratio on average vastly exceeds 10%, with averages of about 60% and quite stable over time (Table I). We obtain stock returns (in RMB) and accounting data from Wind info, the leading Chinese financial database that serves more than 90% of the financial enterprises in the Chinese market. The volatility, shown in Table I as a per-month figure, is quite high, but drops in recent years. Finally, we obtain monthly data on China’s real effective rate based on consumer price indices from Datastream. We convert it to the price, in Yuan, of the currency basket; that is, a positive $D$ refers to a rise of the foreign currencies, and exposure should be positive.

The resulting time series of monthly values of $S_t$ is shown in dotted lines in Figure 1. Over the whole period, the Yuan has appreciated, i.e., the export-destination currencies fell, by 22% from an initial level normalized to 1 in July 2005 to 0.78 in December 2013. Because of the exporter’s flexibility to adjust technology and firm size in our model, it is not the absolute level of the exchange rate that determines the firm’s sensitivity to the exchange rate, but the position of the Yuan relative to its recent standard. One determinant of the recent critical rate is the moving average $\bar{S}_t$. In this respect, it is important to note the episode of rapid depreciation of the Yuan between March 2009 ($S = 0.83$) and November 2009 ($S = 0.89$). As can be seen in panel A of Figure 2, the spread $S_t - \bar{S}_t$ first becomes negative, indicating a regime of local appreciation. It reaches its minimum value of $-0.15$ in November 2008, after which the spread shrinks again and even becomes positive.

<table>
<thead>
<tr>
<th>year</th>
<th>The proportion of foreign sales (in %)</th>
<th>Stock returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>2006</td>
<td>60.14</td>
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</tr>
<tr>
<td>2007</td>
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<td>2008</td>
<td>52.94</td>
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<td>2010</td>
<td>59.68</td>
<td>63.73</td>
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<tr>
<td>2011</td>
<td>59.99</td>
<td>67.14</td>
</tr>
<tr>
<td>2012</td>
<td>60.04</td>
<td>64.38</td>
</tr>
</tbody>
</table>
Figure 1. Time series plots of the real effective exchange rate $S_t$ (full line, right side axis), its $L = 18$ months moving average $\bar{S}_t$ (dashed line, right side axis) and their spread (dotted line, left side axis).

Figure 2. Time series plots of the threshold value $S_{x,t}$ for various values of the barrier $B$ and the percentage number of months for which the option to export has no value.

Notes: As specified in Equation (14), $S_{x,t} = \text{Min}(\bar{S}_t - B, S_{x,t-1})$, with $\bar{S}_t := \sum_{l=1}^{L} S_{t-l} / L$, $S_{x,0} = \max \{ S_0 - B, 0 \}$. Panel A shows, for $L = 18$, the time series of $S_{x,t}$ for various values of $B$. Panel B reports, for all values of $B \in [0, 1]$, the percentage number of months for which the exchange rate $S_t$ is falls below the critical rate $S_{x,t}$. 

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between March and November 2010, after which it returns to negative values. At 
$L = 18$ the autocorrelation in the deviation (shown as the full line) is obvious, but without reaching the levels one associates with martingales. Autocorrelation in the gap relative to the critical rate, however, is much higher: the steady slide of the moving average until the Fall of 2010 is deemed to lead to trigger continuous technological change, leading to rapidly widening safety margins $S_t - \bar{S}_t$ when the Yuan starts recovering in late 2009. Then the exporters can take it easier for about 3 years—until 2013, when the moving average reaches lows not seen since end 2010 and investments are deemed to resume. Time series plots of $\bar{S}_t$ for various levels of $B$, the minimum safety margin, are plotted in panel A of Figure 2.

In our model, the option to export has value whenever $S_t$ exceeds a threshold value $S_t$ that depends non-linearly on the moving average indicator $\bar{S}_t$ and the value of the barrier $B$. The latter is estimated. However, note that, in our sample, not all values of $B$ are acceptable: our selection criterion is that the firm be an active exporter, so that in this sample the option to export must have always value. Panel B of Figure 2 accordingly plots also the percentage of months for which $S_t$ falls below $S_t$. It is only when $B \geq 0.14$ that the option to export has always value. We impose this bound constraint in the estimation.

4.2 Test Equations and Estimation Procedure

We start from our most general one-regime equation, including both the linear exposure term and the interaction variable that come from the Taylor expansion, as well as the market-return:

$$M_1 : r_t = \alpha + \beta r_{mt} + \lambda_x s_t + \delta_x [2(S_{t-1} - \bar{S}_{x,t-1})s_t + \epsilon_t]$$

Recall that, compared with the benchmark loglinear models of Dumas (1978) and Jorion (1990), the proposed exposure model in Equation (16) has a few distinctive features. First, exposures are non-constant; more precisely, they are positive linear functions of the moneyness of the trading activities, defined as the real exchange rate in excess of the critical value; (ii) the firm value is convex in the real exchange rate; and (iii) $\lambda_x$ should be small, as it should estimate the exposure of the domestic fall-back position; most of the action should come from $\delta_x$. Given our choice for a quadratic model of the dependence of firm value $V$ on the exchange rate $S$, the first two are indistinguishable: for a quadratic, convexity in moneyness means that $\delta_x$ is positive, and our expression for the elasticity in Expression (15) then ensures that the elasticity rises in moneyness. So much of our interest is in the presence of positive deltas for our sample of Chinese committed exporters.

However, the equation is plagued by quasi-multicollinearity. The correlation between the linear exposure regressor ($s_t$) and the second order exposure variable $2(S_{t-1} - \bar{S}_{x,t-1})s_t$ is already 85% if we set $B = 0.14$ (the minimum level that ensures nonzero moneyness, a requirement for active exporters; see again panel B of Figure 2), and rises further the higher $B$. At $B = 0.23$, for instance, the correlation is already 0.95, and our estimates of $B$ are typically even higher than 0.23.

There is an intrinsic interest in the two special cases that the above is nesting: the seminal Jorion regression (where $\delta_x = 0$) and the special case of our model where $\lambda_x = 0$ (domestic activities have no exposure). An additional reason for considering these special cases is, of course, the extreme correlation between the regressors. As the general equation is no
good at sorting out the separate contributions of its two regressors, we can get some clues by looking at regressions where only one of the two currency variables is present:

\[ M_2 : \ r_t = \alpha + \beta r_{mt,t} + \lambda x s_t + \epsilon_t, \]  

\[ M_3 : \ r_t = \alpha + \beta r_{mt,t} + \delta T \{ 2(S_{t-1,t} - S_{x,t-1}) \} s_t + \epsilon_t. \]

As noted in the introduction, the Jorion model should have an exposure above unity if the real-option logic applies, that is, if \( V \) is convex in \( S \). The challenger model merely predicts a positive delta, as we just argued. In either regression the estimates should be interpreted with reservations, of course: we see no good way of knowing to what extent the selected regressor is picking up effects really generated by the other. What we do get is an idea of which model does best, which then suggests that this one should not be omitted from the theory and should be preferred by users.

Finally, we check the robustness of all our findings to omitting the market-return regressor:

\[ M_1' : \ r_t = \alpha + \lambda x s_t + \delta T \{ 2(S_{t-1,t} - S_{x,t-1}) \} s_t + \epsilon_t, \]

\[ M_2' : \ r_t = \alpha + \lambda x s_t + \epsilon_t, \]

\[ M_3' : \ r_t = \alpha + \delta T \{ 2(S_{t-1,t} - S_{x,t-1}) \} s_t + \epsilon_t. \]

The results for the first equation suffer from the same quasi-multicollinearity as its counterpart with \( r_{mt,t} \). In the other two regressions we expect more instances of significance.

The estimation results for all these models are summarized in Tables II and III. Estimation is by nonlinear least squares and is done separately for each firm. A computationally convenient grid search is used to estimate the models that depend on the threshold parameter \( B \). The procedure is based on the observation that, given \( B \), the model is linear in the regression coefficients, so one can concentrate out all other regression parameters from the expression of the residual sum of squares. The base case in Panel A is when \( S_{x,t} \) is defined using the 18-months moving average and \( B \in [0.14; 1] \). To interpret this, recall that the exchange rate level is normalized to unity at the beginning of the sample period, and ends at 0.78. So the upper bound \( B \leq 1 \) allows the critical rate to be zero for most of the time. The lower bound of \( B \geq 0.14 \), it may be recalled, is the minimum needed to guarantee positive moneyness throughout the sample. For robustness, panels B and C then study the impact of estimation error in \( B \) by redoing the estimation constraining \( B \) to lie on two more narrow intervals, \([0.5; 0.75]\) and \([0.75; 1]\).

Since \( S_{x,t} \) is a non-differentiable function of \( B \), standard errors accounting for the uncertainty in the barrier estimation \( B \) are not readily available. Moreover, in light of the work

13 Convexity should already hold for the unlevered firm. If the firm is partly debt-financed, the leverage factor should further boost the exposure.

14 We also estimated the models \( M_1 - M_3' \) by least squares panel regression allowing for firm fixed effects. Except for the most general models \( M_1 \) and \( M_1' \) suffering from quasi-multicollinearity between the linear and quadratic exchange rate exposure variables, the exchange rate exposure coefficients are all highly significant and have the expected positive sign. The assumption of homogeneity in the parameters is intuitively hard to accept. The rejection of homogeneity in the exposure coefficients and threshold parameter is also clear from the goodness of fit statistics, showing for example a \( R^2 \) of 2.2% for model \( M_2 \) under the homogeneity parameter assumption versus an average \( R^2 \) of 13% in the equation by equation regression approach.
Table II. Estimation results for models of the exchange rate exposure of 102 Chinese exporters

This table shows the 1% trimmed average coefficient estimates and asymptotic Newey–West standard errors (assuming $B$ fixed), together with the frequency of significantly positive and negative coefficients at the 5% and 10% level, and the average $R^2$. Sensitivity to the value of $B$ is illustrated by doing the analysis for $B \in [0.14; 1]$, $B \in [0.50; 0.75]$, and $B \in [0.75; 1]$. Results are presented for $L = 18$. Model $M_3$ (respectively, $M'_3$) is the recommended extension of the linear exposure model $M_2$ (respectively, $M'_2$). The more general models $M_1$ and $M'_1$ suffer from quasi-multicollinearity.

<table>
<thead>
<tr>
<th>Panel A: $B \in [0.14; 1]$</th>
<th>Panel B: $B \in [0.50; 0.75]$</th>
<th>Panel C: $B \in [0.75; 1]$</th>
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<tr>
<td>$B$</td>
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<td>$\sigma_B$</td>
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of Chan (1993) and Hansen (2000) on the super-fast convergence of the least squares estimate of the threshold parameter in threshold models, it is likely that our estimate of $B$ also has a non-standard asymptotic distribution and converges at a faster rate than the slope parameters. In fact, as can be seen in panel A of Figure 2, any given value of $B$ automatically generates a times series of $S_{x,t}$, which is piecewise constant for often protracted subperiods. As such, the estimate of $B$ takes the form of a change-point estimate and therefore has a non-standard distribution, while the least squares estimates of the remaining parameters can be expected to have the usual asymptotic standard errors. The latter follows from the observation that, given $B$, the model is linear in $a, \beta, k_x, \delta_x$, which is the case for which Hansen (2000) showed that the standard errors of the least squares estimates of the slope parameters are asymptotically independent of the estimation of the threshold parameter. Given the conjectured fast convergence of the least squares estimate of $B$, we thus follow Hansen (2000) and report the usual Newey–West standard errors for the slope parameters only, ignoring the estimation uncertainty in the barrier parameter $B$.

Table III. Impact of the window length $L$ on the estimation results for models of exchange rate exposure of 102 Chinese exporters

This table shows the 1% trimmed average coefficient estimates and asymptotic Newey–West standard errors (assuming $B$ fixed), together with the frequency of significantly positive and negative coefficients at the 5% and 10% level, and the average $R^2$. Sensitivity to the value of $L$ is illustrated by doing the analysis for $L = 9$ and $L = 36$. 

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<th>$\overline{\pi}$</th>
<th>$\lambda_x$</th>
<th>$\delta_x$</th>
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4.3 Results

We start the discussion of the estimation results in Tables II and III with the most general model, $M_1$. Since the estimation of this model is hampered by the similarity of its linear and quadratic exchange rate exposure regressors, the approach that we finally will recommend is to drop the linear exposure term and use the model $M_3$.

The $R^2$ of model $M_1$ is 15%, but the only variable that is systematically significant is the market return. The number of significant $\lambda_x$ and $\delta_x$ coefficients is about what one would expect on the basis of chance under the null of no exposure; for instance, at 5% significance we see three positive $\lambda_x$s and two negative ones, and three positive and three negative $\delta_x$s. Such a pattern of insignificance is what one would expect given the extreme similarity of these regressors.

Subject to the caveats voiced before we accordingly turn to the two special cases nested in $M_1$: the standard model $M_2$ where $\delta_x$ is set at zero, and the quadratic model $M_3$ with $\lambda_x$ set at zero. We first verify for each of the equations $M_2$ and $M_3$ whether the relatively small drop in the $R^2$ (from 0.15 for $M_1$ to 0.13 for $M_2$ and 0.14 for $M_3$) is significant. The $F$-test indicates that for both the $M_1$ versus $M_2$ and $M_1$ versus $M_3$ model comparisons, the loss in explanatory power is only significant for 2 (5) of the 102 Chinese exporters at a 5% (10%) significance level. So the models are too similar to allow a choice on the basis of standard residual sum of squares diagnostics. Looking at other indicators, though, the quadratic model $M_3$ with $\lambda_x$ set at zero does a better job. For 42 out of the 102 Chinese exporters the sample coefficient $\delta_x$ on the second order term $2(S_{t-1} - S_{x,t-1})s_t$ is positive and significant at a 90% confidence level, against just four cases of significantly negative estimates. This result is to be compared first with the $M_2$ model of Jorion (1990) where we have a positive and significant linear exposure term for only 26 out of the 102 Chinese exporters. At the 95% confidence level we have even stronger results: the nonlinear model delivers twenty-nine significantly positive estimates, against just eleven for the linear model. Thus, if one has to choose, the quadratic model seems to be a better description of the conditional relationship between stock returns and the exchange rate in the sense that it allows much more precise estimates of exposure.

As noted, in $M_3$ and $M_2$, the market return may pick up part of the firm’s exposure to the exchange rate. In line with this, dropping the market return as a regressor leads effectively to a substantial increase in the number of significant and positive exposure coefficients. For the Dumas (1978) model with only the linear exposure term the number of significant and positive coefficients $\lambda_x$ increases from 26 to 70 compared with the Jorion (1990) model. For our base model, the number of significant and positive coefficients $\delta_x$ in $M_2$ is 65, up from 42 in $M_3$. As before, the linear model still underperforms in terms of $R^2$, although the difference remains small.

The cost of omitting the market return as a regressor is that the average $R^2$ drops to less than half the original level. Unsurprisingly, this loss in explanatory power is significant: 93 (96) out of the 102 Chinese exporters return an $F$-value beyond the 5% (10%) significance level. If the objective of the estimation is to obtain a good explanation of stock returns, the market return should be retained. Other considerations matter too: for applications in finance the decision may be different depending on whether the idea is to test an asset-pricing model (where we need both a market- and a currency-exposure), versus to demonstrate the existence and/or non-linearity of exposure per se, regardless of how much of that is captured by the exposure to the market return, $\beta_t r_{m,t}$. A hedger, finally, should choose depending on whether or not an index futures contract is being added as a hedge.
We conclude with a discussion of the size and impact of $B$ and $L$, i.e. (i) the margin between the moving average of past rates and the critical rate for a given technology and (ii) the size of the moving-average window that determines how fast the critical exchange rate moves.

Panels B and C in Table II show the results for $B$. First of all, we see that the average value of $B$ for the 102 exporters is a hefty 0.56. This makes moneyness rather variable over time, unlike in earlier work, where moneyness is treated as a constant or is otherwise absent. This matters because it induces substantial variation in the relative exposure, $2(S_{t-1} - \bar{S}_{x_{t-1}})\delta_x$. As shown in Figure 3, for $B = 0.56$ the initial value of the proportionality factor $2(S_{t-1} - \bar{S}_{x_{t-1}})$ is 1.13 (July 2005) and steadily rises to a local peak of 1.17 in June 2006, only to fall back steeply to a low of 0.83 in November 2008. The sharp drop at the end corresponds to the episode of rapid depreciation of the foreign currencies in 2008–9. For the next 2 years the currencies did not move in any major way, but the preceding drop leads to a falling moving average and thus a rising moneyness: the factor $2(S_{t-1} - \bar{S}_{x_{t-1}})$ rises to a maximum of 1.23 in October 2010. Then the foreign currencies resume their downward path, so that $2(S_{t-1} - \bar{S}_{x_{t-1}})$ reverts to more average levels toward the end of the sample. The fact that our deltas are more significant than are exposures obtained via a constant-elasticity model tells us that the variation of $2(S_{t-1} - \bar{S}_{x_{t-1}})$ is quite relevant and should not be ignored.

Figure 3. Time series plots of moneyness $(S_{t-1} - \bar{S}_{x_{t-1}})$ of the export option for Chinese firms when $B = 0.56$ and $L = 18$. 

We conclude with a discussion of the size and impact of $B$ and $L$, i.e. (i) the margin between the moving average of past rates and the critical rate for a given technology and (ii) the size of the moving-average window that determines how fast the critical exchange rate moves.

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Apart from the size and economic relevance of $B$, another issue is to what extent the restrictions on its estimate affect the main findings. First we shift the search zone for $B$ from $[0.14; 1]$ (panel A) to $[0.75; 1]$ (panel B), an interval that does not include the 0.56 estimate for the average from the less constrained search. What we see is that the $R^2$ of the model $M_3$ stays at 14% but the number of positive and significant coefficients decreases from 42 to 34. So getting the level of $B$ right is material. Equally, the less restricted search seems to have been inefficient in allowing some excessively egregious estimates: when in panel C the search zone is confined to $[0.50; 0.75]$ instead of $[0.14; 1.00]$, the number of significant deltas rises from 42 to 47. All this seems to indicate that while the grid search over a wide interval for $B$ remains useful for determining the appropriate threshold value, the model estimates would benefit from more precise a priori information on $B$. However, even rather poorly chosen intervals leave unaffected the conclusion that far more deltas are significantly positive than what one expects just on the basis of chance while very few estimates are significantly negative.

The barrier value $B$ is not the only nuisance parameter required to compute the critical rate $S_{x,t}$; we also need the window length $L$. Our main results in Table II are obtained for $L = 18$, i.e., the longest window length for which the moving average $\overline{S}_t$ of the $L$ lagged values of the exchange rate $S_t$ for which the augmented Dickey–Fuller test detects no unit root at a 95% confidence level. Table III shows results for a shorter ($L = 9$) and longer ($L = 36$) window choice. Overall, these robustness checks are in line with the previous results. For $L = 9$ and 36, a positive value of $\delta_x$ is only rejected for two or five firms, respectively, out of the 102 Chinese exporters in the model with market return and eight in the model without the market return; the number of significantly positive deltas, in contrast, remains much higher, as before. Also the explanatory power of the models in terms of $R^2$ for $L = 9$ and $L = 36$ is similar as for $L = 18$.

5. Discussion and Conclusion

One would intuitively expect that the value of many exporting firms positively depends on the exchange rate. The exact nature of this dependence is still an open question. Often, log-linearity is imposed leading to a linear relationship between the returns on firm value and the exchange rate returns. If the elasticity is below unity, this model is at odds with the real options logic, which predicts a convex relation between firm value and the exchange rate. And even if the estimate turns out to exceed unity, as it does in our sample, the model still misses its contingency upon the exchange rate. The option to export, given a technology, should already lose its value at some critical exchange rate strictly above zero; it follows logically that the exchange rate elasticity should similarly have petered out long before the exchange rate reaches zero. In general, then, the elasticity is not constant.

In this article, we develop a model that allows for both these desirable features, by considering a second-order Taylor expansion of the firm value to the right of the exchange rate that serves as the threshold value, that is, the critical rate as of which the firm’s option to engage in international activities becomes valuable. We test this model on a sample of systematic Chinese exporters over the period July 2005–December 2013 and find substantial empirical evidence in favor of the model in which exposures are dynamic and where, in line with the real-option nature of the exporting business, the exposure increases when the value of the export option, as measured by its moneyness, increases. Depending on whether the market return is included in the test regression, a significant time-varying exchange rate
elasticity is detected for around 40% or 65% of the Chinese exporters in our sample, while instances of significant concavity are quite rare. The strength of the evidence is probably traceable to some unique features of the data set. Whereas the presence of foreign sales in a US multinational does not necessarily mean that this firm is predominantly an exporter, our Chinese firms unambiguously are, with foreign-sales ratios of 60% on average. Also, our Chinese firms do not have any access to forward hedges. So two potential confounding features, operational and financial hedging, are absent.

We conclude with a few methodological suggestions for further research. In our work, we just use stock-price and exchange-rate data. Expanding the information set with firm-level data on exports and fixed assets should lead to a richer and more powerful model. But such data are often only available at a quarterly or lower frequency, if at all. In that sense the choice for a particular set of numbers is a matter of trading off richness of data against availability and sample size.

Another aspect that could be improved is the exchange-rate side. Following a long tradition we worked with an index, and a one-size-fits-all index at that; our focus was on the modeling side and the taxonomy, both of which would be greatly complicated if multiple individual exchange rates had been chosen. Still, there is a clear case in favor of using a few key bilateral rates rather than a single, fixed index, or at least constructing a firm-specific currency index that does take into account the firm’s own varying trade patterns.

Finally, our analysis focuses on Chinese exporters. In earlier estimation rounds we worked also with US multinationals, allowing for both export and import terms with different exposures and critical exchange rates, yielding models with up to three regimes and six free parameters describing exposure. One risk with such a rich parameterization is overfitting. Another issue in such samples is that Western multinationals do hedge, both financially and operationally, which may destroy much of the features predicted here. Further research is needed to develop models and estimation methods that can describe the dynamics in the exposure of such complex firms.

References


